

# PLANCK-LENGTH PHENOMENOLOGY<sup>1</sup>

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## Abstract

This author’s recent proposal of interferometric tests of Planck-scale-related properties of space-time is here revisited from a strictly phenomenological viewpoint. The results announced previously are rederived using elementary dimensional considerations. The dimensional analysis is then extended to the other two classes of experiments (observations of neutral kaons at particle accelerators and observations of the gamma rays we detect from distant astrophysical sources) which have been recently considered as opportunities to explore “foamy” properties of space-time. The emerging picture suggests that there is an objective and intuitive way to connect the sensitivities of these three experiments with the Planck length. While in previous studies the emphasis was always on some quantum-gravity scenario and the analysis was always primarily aimed at showing that the chosen scenario would leave a trace in a certain class of doable experiments, the analysis here reported takes as starting point the experiments and, by relating in a direct quantitative way the sensitivities to the Planck length, provides a model-independent description of the status of Planck-length phenomenology.

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The realization that the magnitude of quantum-gravity effects is likely to be suppressed by some power of the minute Planck length  $L_p$  ( $\sim 10^{-35}m$ ) has traditionally led to the expectation (see, *e.g.*, Ref. [1]) that it might be impossible to obtain pertinent experimental information. Some recent studies [2, 3, 4] (also see follow-up work in Refs. [5, 6, 7]) have however suggested that certain experiments are finally reaching a level of sensitivity such that some quantum-gravity effects could in fact be seen, at least if the magnitude of these effects is estimated using the most “optimistic” (here optimism equals large estimated effects) theoretical quantum-gravity scenarios. These previous studies were focused on one or another quantum-gravity scenario and the analysis was primarily aimed at showing that the chosen scenario would leave a trace in a certain class of doable experiments. This note examines the proposals [2, 3, 4] (focusing primarily on this author’s recent [4]) with the objective of extracting a model-independent characterization of the experimental sensitivities that are being reached. It will be shown that these sensitivities can be expressed very intuitively in terms of the Planck length, in ways that appear to justify a description of the present phase of exploration of space-time properties as the first steps of “Planck-length phenomenology”.<sup>2</sup>

The interferometric studies proposed in Ref. [4] were motivated by the observation that the sensitivity of modern interferometers, even the 40-meter interferometer already in operation at Caltech [12], is such that one can already rule out [4, 8] the possibility of fluctuations in the length  $L$  of the arms of these interferometers that are of Planck-length magnitude and occur at a rate of one per Planck time. Of course, Planck-length-related sensitivity to distance fluctuations can be potentially significant for quantum gravity. The interplay between gravitation and quantum mechanics could plausibly result in a picture of space-time which at short distances is somewhat fuzzy and indeed involves distance fluctuations. This is for example true of most quantum-gravity approaches hosting some form of “space-time foam” [13, 14]. The observation reported in Refs. [4, 8] indicates that the sensitivity of interferometers is finally reaching a point where it can be related rather naturally to the Planck length; however, the analysis that allows a comparison between the data and the mentioned random-walk scheme of distance fluctuations is quite involved and some of the intuition for the significance of these experimental achievements can be lost through the lines of the derivation. With some algebra one shows [4, 8] that random-walk fluctuations of Planck length magnitude occurring at a rate of one per Planck time correspond to a “strain noise power spectrum”<sup>3</sup>  $\rho_h = cL_p L^{-2} f^{-2}$ , and then one observes [4] that for  $L \sim 40m$  and  $f \sim 500Hz$  this formula leads to a prediction that is inconsistent with the noise-level measurements [12] performed by the Caltech 40-meter.

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<sup>2</sup>It is here proposed to adopt the denomination “Planck-length phenomenology”, instead of the previously used [8] “quantum-gravity phenomenology”, as a way to emphasize that the corresponding area of research is motivated by the fact that some experiments recently managed to achieve Planck-length-related sensitivities, more than by any detailed conjecture on the short-distance structure of space-time. While in this note the focus is on experiments relevant for space-time-foam studies, Planck-length phenomenology is also active on other aspects of the interplay between gravitation and quantum mechanics (see, *e.g.*, Refs. [9, 10, 11, 8] and references therein).

<sup>3</sup>As discussed rather pedagogically in Ref. [15], the power spectrum  $\rho_h(f)$  of the strain noise that would be induced in the length of the arms of an interferometer is the most convenient way to characterize models of distance fluctuations. In fact, the strain  $h \equiv \Delta L/L$  is a measure of the displacement  $\Delta L$  in a given distance  $L$ , and the associated strain noise power spectrum is a spectral function that depends only on the frequency of observation and contains the most significant information on the nature of the distance fluctuations (such as the mean square deviation, which can be obtained as the integral of the power spectrum over the bandwidth of observation [15]).

An exercise that might contribute to the development of a more intuitive understanding of these results is the one of realizing that a simple dimensional analysis, without any detailed computation, suffices to show that a formula of the type  $\rho_h = cL_p\Lambda^{-2}f^{-2}$ , with  $\Lambda$  a length scale to be determined, must hold for any model with distance fluctuations that are of random-walk type and induce effects that are linear in the Planck length. This follows from taking into account that  $\rho_h$  carries dimensions of  $\text{Hz}^{-1}$  and that there is a general relation [16] between stochastic processes of random-walk type and  $f^{-2}$  behaviour of the power spectrum. It is therefore proper to state that the theoretical aspects of the analysis reported in Refs. [4, 8] are only necessary in order to fix the value of  $\Lambda$ : they show that  $\Lambda = L$  for a random-walk scheme with Planck-length fluctuations in the length  $L$  of the arms of the interferometer occurring at a rate of one per Planck time. The phenomenological aspects of the analysis reported in Refs. [4, 8] show that the data reported in Ref. [12] imply that the prediction  $\rho_h = cL_p\Lambda^{-2}f^{-2}$  is inconsistent with available experimental data unless  $\Lambda \gg L$  (which is a rather significant bound since  $L$  is the largest physical length scale in the experimental context).

This more intuitive reinterpretation of the results reported by this author in Refs. [4, 8] can be cast into an even more general perspective by noticing that there is a simple sense in which the sensitivity of interferometers is very clearly at the level of effects linear in the Planck length. In fact, modern interferometers are rapidly approaching sensitivity levels corresponding to strain noise power spectrum of order  $\rho_h \sim 10^{-44}\text{Hz}^{-1}$  (the sensitivity which the LIGO/VIRGO generation [17, 18] of interferometers is planning to achieve, at least in a relatively narrow bandwidth, already in their “first phase of operation”) and this sensitivity level is quite naturally expressed as the ratio between the Planck length and the speed-of-light constant  $L_p/c \sim 5 \cdot 10^{-44}\text{Hz}^{-1}$  (*i.e.* the “Planck time”).

It is somewhat amusing to notice that these interferometers have been motivated to reach sensitivities in the neighborhood of  $10^{-44}\text{Hz}^{-1}$  because their primary objective is the discovery of the classical-physics phenomenon of gravity waves, predicted by Einstein’s general relativity, and it just happens to be the case that the relevant classical-physics studies have led to the conclusion that a sensitivity somewhere between  $10^{-44}\text{Hz}^{-1}$  and  $10^{-46}\text{Hz}^{-1}$  is needed for the discovery of classical gravity waves. It is a remarkable numerical accident that the result of these classical-physics studies (in which, in particular, the magnitude of the effects has a strong dependence on the expected distance between our detectors and the relevant astrophysical sources of gravity waves) pointed us toward a sensitivity level which I am now observing to be also naturally associated with the intrinsically quantum scale  $L_p/c$ .

One other reason of interest in the use of interferometers in space-time foam studies is that quantum-gravity noise would be a property of the apparatus. This might be the first chance to explore whether indeed there are drastically new phenomena [8] associated with the quantum properties of the gravitational degrees of freedom of an apparatus. Interferometers are so sensitive that already the quantum properties of non-gravitational degrees of freedom of the apparatus play a non-trivial role [15], and it appears legitimate to wonder whether quantum properties of the gravitational degrees of freedom of the apparatus are also significant. Such properties, when described operatively as in Ref. [8], are the first operatively-defined properties of space-time foam (which is otherwise an intuitive but abstract concept).

Considering now the other two quantum-gravity experiments, the study of foam-deformed neutral-kaon dynamics using particle accelerators [2, 5] and the study of foam-deformed photon propagation using data on the gamma rays we detect from distant astrophysical sources [3, 6], it is important to observe that an intuitive understanding based on dimensional analysis in terms of fundamental constants is also possible. Tests

of CPT symmetry in the neutral kaon system are now [2, 5], at the sensitivity level  $|M_{K_0} - M_{\bar{K}_0}|/M_{K_0} < 10^{-18}$ . Even without a specific quantum-gravity model it is easy to realize that it would not be surprising to find some sort of CPT-violating effects in quantum gravity (which in particular might involve some form of non-locality, quantum decoherence, and other “CPT-theorem-hostile” features) and a straightforward dimensional analysis allows to estimate that the magnitude of such effects in the neutral kaon system, if linear in  $L_p$ , would be naturally set by  $L_p c M_{K_0} / \hbar \sim 5 \cdot 10^{-19}$ . Also in this case of neutral-kaon studies of CPT-violating effects it is therefore proper to state that the sensitivities being reached correspond to the natural strength of effects which could emerge within the hypothesis of linearity in the Planck length.

Tests of Poincaré symmetries using data on the gamma rays we detect from distant astrophysical sources are rapidly becoming more and more sensitive with the improvement of the techniques of signal analysis and source-distance measurement. In particular, the limits on the possibility of an energy-dependence of the time of arrival of photons of different energies emitted simultaneously by a distant astrophysical source are now [3, 6], depending on the particular phenomenon being studied, somewhere between the levels  $\Delta T/T \sim 10^{-22}$ , which is within the reach [3] of gamma-ray-burst experiments analyzing photons with energies typically in the range  $100 \text{ KeV} < E < 1 \text{ MeV}$ , and  $\Delta T/T \sim 10^{-16}$ , which is within the reach [3, 6] of experiments analyzing the very-high-energy (multi-TeV) photons we receive from other types of astrophysical sources. (Here the intuitive notations  $T$  and  $\Delta T$  have been adopted respectively for the overall time of travel and the difference in the times of arrival of two photons within the signal.) Even without a specific quantum-gravity model it is easy to realize that it would not be surprising to find some form of Poincaré-violating effects in quantum gravity (as discussed rather pedagogically in Ref. [8, 19, 20, 21], these are indeed quite natural because of the element of discreteness introduced by the Planck length); these in turn would affect the structure of the dispersion relations which characterize the laws of particle propagation, quite possibly leading to an energy-dependence of these laws [3, 19, 21]. Working again in the hypothesis of effects linear in the Planck length, and taking into account the fact that this particular effect should manifest a dependence on the energy difference  $\Delta E$  between the two photons whose travel times are being compared, it is easy to identify the dimensionless ratio that could set the magnitude of this quantum-gravity effect:  $\Delta T/T \sim L_p \Delta E / (c\hbar)$ . For  $\Delta E \sim 1 \text{ MeV}$ , as found in some gamma-ray-burst data, this would lead to the estimate  $\Delta T/T \sim 10^{-22}$  which is just of the order of the “ $\Delta T/T$  sensitivity” being reached [3] by gamma-ray-burst experiments.

The model-independent characterization of the sensitivities of the three mentioned “quantum-gravity experiments”, which is the main objective of the present note, is now complete. It is worth emphasizing again that the three experiments all happen to have sensitivities corresponding to the estimates one obtains assuming that the magnitudes of the new effects depend linearly on the Planck length. This coincidence is even more remarkable in light of the fact that not long ago the sensitivities of the three experiments were several orders of magnitude away from the present levels and they were all different from one another. The fact that these experimental sensitivities have improved so much and now happen to cross together the significant milestone represented by effects linearly suppressed by the Planck length gives added significance to the present phase of Planck-length phenomenology.

One may wonder what makes these three experiments so special that they managed to reach such an extraordinary level of sensitivity. This is easily understood as being mostly related to the fact that the three experiments are all structured in ways that

render them capable of detecting some “collective effects”, effects which are the end result of a large number of minute quantum-gravity effects. The gloomy forecasts for Planck-length phenomenology which one finds in traditional quantum-gravity reviews are based on the observation that under ordinary conditions the direct detection of a single quantum-gravity effect would be well beyond our capabilities, if the magnitude of the effect is suppressed by the smallness of the Planck length (*e.g.*, in particle-physics contexts the graviton contribution to ordinary scattering processes is completely negligible). However, small effects can become observable when the experimental setup is such that a very large number of the small effects is somehow put together. This later possibility is not unknown to the particle-physics community, since it has been exploited [22] in the context of proton-decay experiments, where stringent bounds are achieved by monitoring a very large number of protons. An interferometer with bandwidth roughly centered at  $100\text{Hz}$  is collecting high-quality data over a time scale of order  $10^{-2}s$ , and can therefore in principle be sensitive to the collective effect of an extremely large number of distance fluctuations if these fluctuations occur with a characteristic rate of one per Planck time ( $\sim 5 \cdot 10^{-44}s$ ). For example, in the mentioned random-walk  $L_p$ -linear scheme of distance fluctuations the length scale that most genuinely characterizes the associated noise at  $100\text{Hz}$  is  $\sqrt{L_p 10^{-2}s} \sim 10^{-14}m \gg L_p$ . Similarly, photons reaching us from distant astrophysical sources have traveled for a very long time and can in principle be sensitive to the collective effect of an extremely large number of “interactions” with the space-time foam, if these are relatively frequent. The case of studies of neutral kaons is even more interesting. The effect turns out to be testable because of two “positive factors”: the kaon lifetime, while being significantly shorter than astrophysical propagation times, is still significantly larger than the Planck time (thereby providing an opportunity for sensitivity to the collective effect of a large number of interactions with the space-time foam), and, in addition, the neutral-kaon system hosts a very delicate balance of mass scales, reflected in peculiarly small dimensionless ratios such as  $|M_{K_L} - M_{K_S}|/M_{K_0} \sim 10^{-14}$ , which renders it very sensitive [2, 5] to certain types of new phenomena.

Even though the analysis reported in the present note is strictly phenomenological, this author hopes to have provided a tool that can be useful also for work on the quantum-gravity formalism: while previous phenomenological analyses might have left those working on the formalism uncertain on whether or not these developments could be relevant for their chosen approach, the present model-independent characterization should give a more intuitive picture of the sensitivities. This might in some cases be sufficient for the identification of the elements of the formalism that are needed in order to make the connection with phenomenology. “Canonical/Loop Quantum Gravity” [23] and Superstring Theory might be already mature enough for this type of analyses. A key problem remains the one of developing a plausible measurement theory for these quantum-gravity approaches. In “Canonical/Loop Quantum Gravity” some important open issues<sup>4</sup> with respect to measurement theory have already been discussed in Ref. [25].

Going back to the phenomenological viewpoint for the closing remarks, it is perhaps worth emphasizing that the chances of discovery of new physics by Planck-length phenomenology appear to depend very strongly on whether or not there is in Nature an effect

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<sup>4</sup>One important missing element is a compelling candidate as length observable. This has wide implications for all physical predictions, since most measurement procedures require some distance measurements (as illustrated by the example considered in Ref. [25], some distance measurements naturally intervene at intermediate stages of the procedure, even when the observable in which one is finally interested is not a distance).

whose magnitude goes linearly with the Planck length. The three experiments considered in the present analysis probe space-time in rather complementary ways but all have sensitivities corresponding to linearity in the Planck length. It appears therefore likely that new effects linear in the Planck length would eventually be discovered, while instead the case of effects suppressed quadratically by the Planck length might be beyond our reach.

## References

- [1] C.J. Isham, *Structural issues in quantum gravity*, in *Proceedings of General relativity and gravitation* (Florence 1995).
- [2] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, *Search for violations of quantum mechanics*, Nucl. Phys. B241 (1984) 381.
- [3] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos and S. Sarkar, *Tests of quantum gravity from observations of  $\gamma$ -ray bursts*, astro-ph/9712103, Nature 393 (1998) 763-765.
- [4] G. Amelino-Camelia, *Gravity-wave interferometers as quantum-gravity detectors*, gr-qc/9808029, Nature 398 (1999) 216.
- [5] J. Ellis, J. Lopez, N. Mavromatos, D. Nanopoulos and CPLEAR Collaboration, Phys. Lett. B364 (1995) 239; V.A. Kostelecky and R. Potting, Phys. Rev. D51 (1995) 3923; P. Huet and M.E. Peskin, Nucl. Phys. B434 (1995) 3; F. Benatti and R. Floreanini, Nucl. Phys. B488 (1997) 335.
- [6] B.E. Schaefer, Phys. Rev Lett. 82 (1999) 4964; S.D. Biller *et al*, Phys. Rev. Lett. 83 (1999) 2108; G. Musser, Scientific American, October 1998 issue.
- [7] D.V. Ahluwalia, gr-qc/9903074, Nature 398 (1999) 199; M. Brooks, New Scientist 2191 (1999) 28; A. Campbell-Smith, J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B466 (1999) 11; G. Amelino-Camelia, gr-qc/9903080, Phys. Rev. D62 (2000) 024015; Y.J. Ng and H. van Dam, gr-qc/9906003; Hong-wei Yu and L.H. Ford, gr-qc/0004063.
- [8] G. Amelino-Camelia, *Are we at the dawn of quantum-gravity phenomenology?*, gr-qc/9910089, notes based on lectures given at the XXXV Karpacz Winter School of Theoretical Physics *From Cosmology to Quantum Gravity*, Polanica, Poland, 2-12 February, 1999 (published in the volume entitled “Towards Quantum Gravity”, Springer-Verlag Heidelberg 2000, edited by J. Kowalski-Glikman).
- [9] R. Brustein, M. Gasperini, M. Giovannini and G. Veneziano, Phys. Lett. B361 (1995) 45.
- [10] G.F. Giudice, hep-ph/9912279, *Recent developments in physics beyond the standard model*, talk given at the *19th International Symposium on Lepton and Photon Interactions at High-Energies*, Stanford, California, 9-14 Aug 1999 (to appear in the proceedings).
- [11] G.Z. Adunas, E. Rodriguez-Milla and D.V. Ahluwalia, Phys. Lett. B485 (2000) 215.

- [12] A. Abramovici *et al*, Phys. Lett. A218 (1996) 157.
- [13] J.A. Wheeler, *Relativity, groups and topology*, ed. B.S. and C.M. De Witt (Gordon and Breach, New York, 1963).
- [14] S.W. Hawking, Nuc. Phys. B144 (1978) 349.
- [15] P.R. Saulson, *Fundamentals of interferometric gravitational wave detectors* (World Scientific 1994).
- [16] V. Radeka, IEEE Trans. Nucl. Sci. NS16 (1969) 17; Ann. Rev. Nucl. Part. Sci. 38 (1988) 217.
- [17] A. Abramovici *et al*, Science 256 (1992) 325. [Updated information on expected sensitivity of an advanced phase of the LIGO interferometer can be found at WWW site <http://www.ligo.caltech.edu/~ligo2/>.]
- [18] C. Bradaschia *et al*, Nucl. Instrum. Meth. A289 (1990) 518; B. Caron *et al*, Class. Quantum Grav. 14 (1997) 1461.
- [19] G. 't Hooft, Class. Quant. Grav. 13 (1996) 1023.
- [20] G. Amelino-Camelia, Mod. Phys. Lett. A13 (1998) 1319.
- [21] R. Gambini and J. Pullin, Phys. Rev. D59 (1999) 124021.
- [22] The intuition guiding this author's contributions to the development of Planck-length phenomenology originates from an analogy with the strategy being used in modern proton-decay experiments which was discussed in G. Amelino-Camelia, *SO(10) grandunification model with proton lifetime of the order of  $10^{33}$  years* (Laurea thesis, Facoltà di Fisica dell'Università di Napoli, 1990).
- [23] Recent reviews of this approach can be found in A. Ashtekar, gr-qc/9901023; M. Gaul and C. Rovelli, gr-qc/9910079 (notes based on lectures given by C. Rovelli at the XXXV Karpacz Winter School of Theoretical Physics *From Cosmology to Quantum Gravity*, Polanica, Poland, 2-12 February, 1999); L. Smolin, Physics World 12 (1999) 79.
- [24] A. Ashtekar, C. Rovelli, and L. Smolin, Phys. Rev. Lett. 69 (1992) 237.
- [25] G. Amelino-Camelia, gr-qc/9804063, Mod. Phys. Lett. A13 (1998) 1155; gr-qc/9808047, in *Proceedings of 7th International Colloquium on Quantum Groups and Integrable Systems* (Prague, Czech Republic, 18-20 June 1998).